Pergamon

Journal of Structural Geology, Vol. 17, No. 6, pp. 783 to 791, 1995

0191-8141/95 $9.50 + 0.00

0191-8141(94)00104-9

Superposed deformations by Mohr construction

SUSAN H. TREAGUS

Department of Geology, University of Manchester, Manchester M13 9PL, U.K.

(*Received 19 January 1994; accepted in revised form 15 September 1994*)

Abstract—The Mohr diagram can provide a simple constructional method for superposing various kinds of deformation. Examples are given for superposed pure shears, and various combinations of pure shear, simple shear and area change. Construction methods are developed on a Mohr diagram for reciprocal stretch vs rotation, which is actually a representation of the 'backwards deformation' tensor, **d**. The superposition of a pure shear or a simple shear on an earlier deformation follows elementary principles in Mohr space, enabling the total deformation to be determined simply. More general deformations can be considered as a combination of dilation, pure shear and simple shear (if body rotations are omitted), which are all amenable to the Mohr construction method.

Although confined to problems of two-dimensional superposed deformation, this new method provides a useful alternative to mathematical or computer-based methods to determine the effects of oblique superposition of several deformations. The visual appeal of such graphical constructions should aid in the teaching and understanding of deformation processes.

INTRODUCTION

The structure and fabric of deformed rocks may commonly be the result of two or more deformations. A theoretical analysis of the effects of two superposed deformations would normally be tackled mathematically by matrix multiplication (e.g. Ramsay & Huber 1983, pp. 291–292), and probably solved by computer methods. While this would be the best way of undertaking a large number of calculations and achieving greatest accuracy, such methods do not provide a visual illustration of the processes of transforming one deformed state to another. As a consequence, mathematical methods may not be easy to incorporate into elementary teaching of deformation processes, or offer an easy route to determining the geometrical consequences of superposing two deformations, for a particular field-based example. This paper presents a new method of determining superposed deformations, using Mohr diagrams.

Mohr diagrams provide a family of graphical illustrations of tensors and their operations (see De Paor & Means 1984). Although Mohr diagrams and circles are a familiar tool of structural geology, the Mohr diagram which represents *deformation* is a more recent application. It was first used in the geological literature by Robin (1977) for infinitesimal deformation, and later developed simultaneously by De Paor and Means for finite strain and deformation (Means 1982, 1983, De Paor 1983). These authors recognized that the *stretch tensor* for two dimensions could be represented as a Mohr circle (see also Choi & Hsü 1971), in which the stretch and rotation of any line are given by polar coordinates. A similar representation of the *deformation tensor*, **D**, which includes components of stretch and rigid rotation (in general an asymmetric tensor), is an *off-axis Mohr diagram* (De Paor 1983, Means 1983, Passchier 1988). The deformation tensor, termed **D** in this paper (after Means), is more precisely called the *position gradients tensor*, and called **F** by some workers.

The Mohr diagram representation of the deformation tensor has considerable potential for teaching principles of strain and deformation (e.g. Means 1990, 1992). The components of a deformation matrix are given by Cartesian coordinates of points on the Mohr circle, whereas the stretch and rotation are given by the polar coordinates. Two sign conventions are in current use (see De Paor & Means 1984), each having advantages for particular problems: compare Means (1992) and Simpson & De Paor (1993). In the present paper, I am following a 'First Kind' of angular convention, where angles are shown in their correct sense. This allows easy use of a *Mohr circle pole*: that unique point on the circle to which any other point can be joined, to reproduce the material line represented by the point in its geographic orientation (Cutler & Elliott 1983, Allison 1984, Treagus 1987).

Mohr diagrams for deformation have considerable potential for solving geological problems graphically, as recently illustrated by Passchier (1988, 1990a,b), Passchier & Urai (1988), Treagus (1990), Wallis (1992) and Simpson & De Paor (1993). The present paper will demonstrate how such a Mohr diagram can be used to determine superposed deformations, graphically. This application is one of the tensor operations for Mohr diagrams introduced in general terms by De Paor & Means (1984).

The *Mohr diagram for reciprocal deformation* ('backwards deformation' tensor, **d**) will be used in this paper (see also Treagus 1990). I consider this form of Mohr diagram better suited to represent progressively deformed states in superposed deformation than the 'forwards deformation' (**D**) diagram. However, the principles followed are similar for either form of Mohr diagram, so users already familiar with the **D** diagram might prefer this representation. The reciprocal (**d**) diagram is equivalent to an upside-down form of the **D** diagram, with a change of scale if there is area change during deformation. (See De Paor & Means 1984, figs. 11 and 12, illustrating tensor inversion on a Mohr diagram.) Similarly, users who prefer a Second Kind of Mohr circle convention (see De Paor & Means 1984, Simpson & De Paor 1993) can easily reverse the sign convention in the following diagrams. However, the facility to use the Mohr circle pole for the deformed state will then be less direct.

The reciprocal deformation Mohr diagram allows the progressively *deformed state* to be represented in terms of reciprocal stretches, rotations and angles, and related to real space via the Mohr circle pole. The starting point is a *First Deformation*, and its Mohr diagram in reciprocal form. A known *Second Deformation* is applied, and critical points on the First circle are transposed to make a Mohr circle for the *Combined Deformation*. The procedures will be developed first for superposed pure shears, and then for simple shear transformations and more general superposed deformations.

SUPERPOSED PURE SHEAR

A common type of first deformation assumed for rocks is an irrotational layer-parallel/normal deformation, such as a compactional strain, or a layer-parallel shortening. For this reason, it seems appropriate to begin with a First Deformation which is a symmetric stretch tensor. The simplest possible case—pure shear with no area change—will be considered first. It will be shown later that the following procedures of two-dimensional superposition will work for any First Deformation.

Two superposed pure shears: the Mohr construction method

Consider two oblique pure shears, as sketched in Fig. 1(a). The Mohr construction method developed here requires the external coordinate axes, x_1, x_2, to be chosen parallel to the principal axes of the Second pure shear.

The chosen *First Deformation* is represented on the reciprocal Mohr diagram in Fig. 1(b). In this example, the first strain has principal stretches of 1.43 and 0.7, and because this is equal-area plane strain, the reciprocal stretches are opposite and equivalent. The *material lines* parallel to external coordinate directions x_1 and x_2 (through which the progressive deformation is being viewed), at the end of the First Deformation, will be called m and n throughout the paper. They become m′, n′ after the superposed deformation increment. It is important in the following method not to confuse these with material lines coincidentally parallel to the external reference axes (x_1, x_2) in the final (combined) deformed state. In this first example of superposed pure shears, these lines correspond, but in later more general cases, they do not. The Mohr circle pole in Fig. 1(b) allows m and n to be represented in their geographic orientations with respect to reference axes, x_1, x_2.

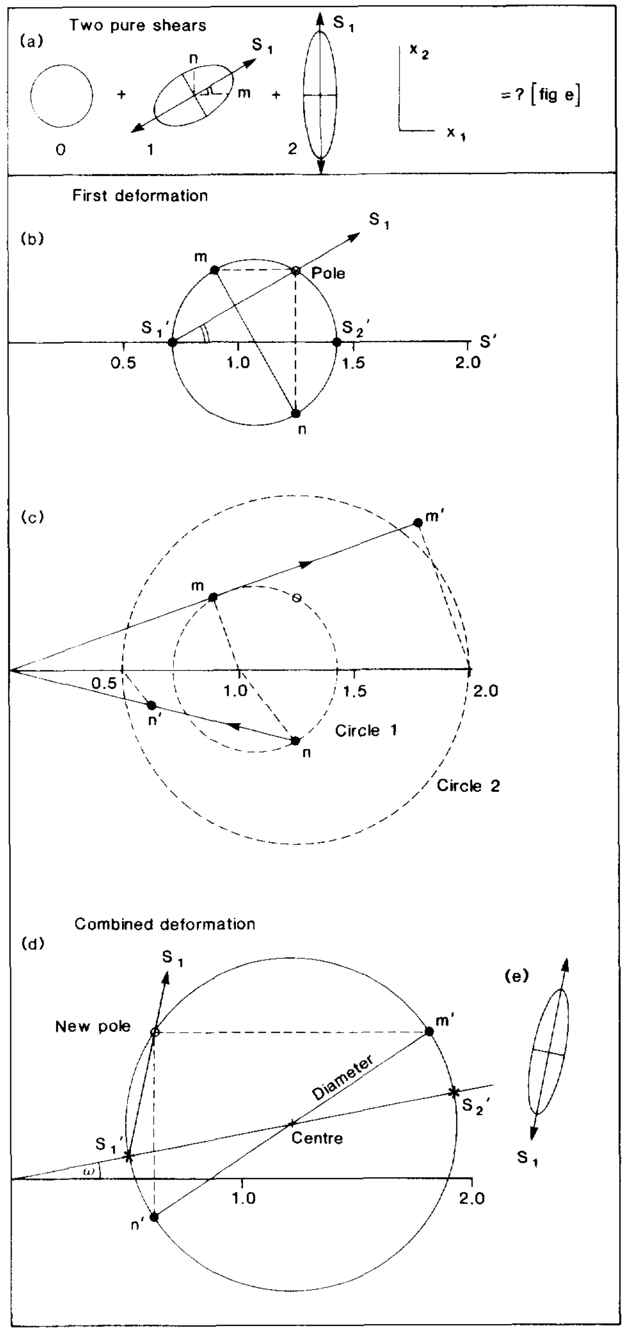

Fig. 1. General method of pure shear superposition. (a) shows the sequence of deformation and strain, schematically. Pure shears 1 and 2 are mutually oblique by 60°. (b) Reciprocal Mohr diagram (**d**) for the First Deformation (1.43, 0.7), also showing m and n in geographic space (dashed lines) by use of the Mohr circle pole. (c) Mohr diagrams for the two pure shears, showing how m and n can be transposed by reciprocal stretching to m′ and n′. Crosses mark the principal reciprocal stretches for the Second Deformation, and dotted lines show the construction method. (d) The Combined Deformation circle is constructed with diameter m′–n′. (e) represents the strain ellipse for the Combined Deformation, completing the sequence in (a). (See text for fuller explanations.)

Points m and n will now be transformed by the pure shear *Second Deformation* to their deformed equivalents, m′, n′. The Second Deformation is shown by circle 2, in Fig. 1(c). In the present example, these principal stretches are 2 and 0.5 (again, equivalent and opposite reciprocal values), so the second increment is a greater strain than the first. Material lines (m, n) parallel to these principal axes will be stretched according to this Second Deformation. The transformation is represented on the Mohr diagram by applying the appropriate *reciprocal principal stretches* by multiplication, to the m and n rays. *Neither ray rotates.* The m ray is stretched by 1/0.5 to point m′, and the n ray by 1/2.0 to n′ (Fig. 1c). Points m and n can be stretched to m′ and n′ entirely by construction, shown in Fig. 1(c) by dotted lines. m and n are joined to the unit-stretch point on the reference axis (1.0), and *similar triangles* constructed from the relevant Second principal stretch points on the axis (crosses). This removes the need for any arithmetic calculation, although it may be less accurate.

By definition, for an irrotational pure shear increment, m′ and n′ remain respectively parallel to the x_1, x_2 axes, so still mutually perpendicular; so they define the diameter of the Combined Mohr circle. The circle is thus drawn, and its principal reciprocal stretches (S_1' and S_2') identified and measured. A ray drawn from the origin to the circle centre indicates the rotational component of the deformation (ω). It has already been noted that m′ remains parallel to x_1, and n′ to x_2; hence, lines drawn in these directions locate the new Mohr circle pole (Fig. 1d). This allows the *Combined Deformation* to be constructed as a strain ellipse in geographic space (Fig. 1e), with the S_1 direction in space parallel to the S_1'-pole line on the Mohr diagram. Alternatively, the S_1 and S_2 directions can be located from their orientations to m′ or n′ on the final Mohr circle, according to conventional Mohr circle methods.

This example of two superposed pure shears is equivalent to 'multiplication by a diagonal tensor', described and illustrated in De Paor & Means (1984, fig. 11). Other examples will now be given, using the graphical method described above.

Other examples of superposed pure shears

Figure 2 illustrates the effects of an equivalent superposed strain in three different orientations. The First Deformation is identical to that in Fig. 1; i.e. pure shear with principal stretches 1.43 and 0.7. In this case, the Second Deformation is an *equal strain increment*, with principal stretches (1.43, 0.7) oriented at (a) 60°, (b) 45° and (c) 30° counterclockwise to the First stretch. The 60° case is the same orientation of superposed deformation as shown in Fig. 1, although the Second Deformation is less intense. In each case in Fig. 2, the Combined Deformation is constructed on a single Mohr diagram.

It is clear that as the angle between the first and second principal stretch directions decreases, the combined deformation increases. In all cases, the combined deformation is an 'off axis' Mohr diagram, which immediately illustrates that the product of two oblique pure shears (each a symmetric deformation tensor) is an asymmetric tensor with components of stretch plus rotation. The rotational component is 5–7° for these three examples, and is proportionally more significant, where the two deformations are strongly oblique. For any combination of two pure shears, such as illustrated in Fig. 2, there will be a value of obliquity which gives rise to a maximum rotation component, just as the two orientations of coaxial superposition give rise to the maximum and minimum stretch component.

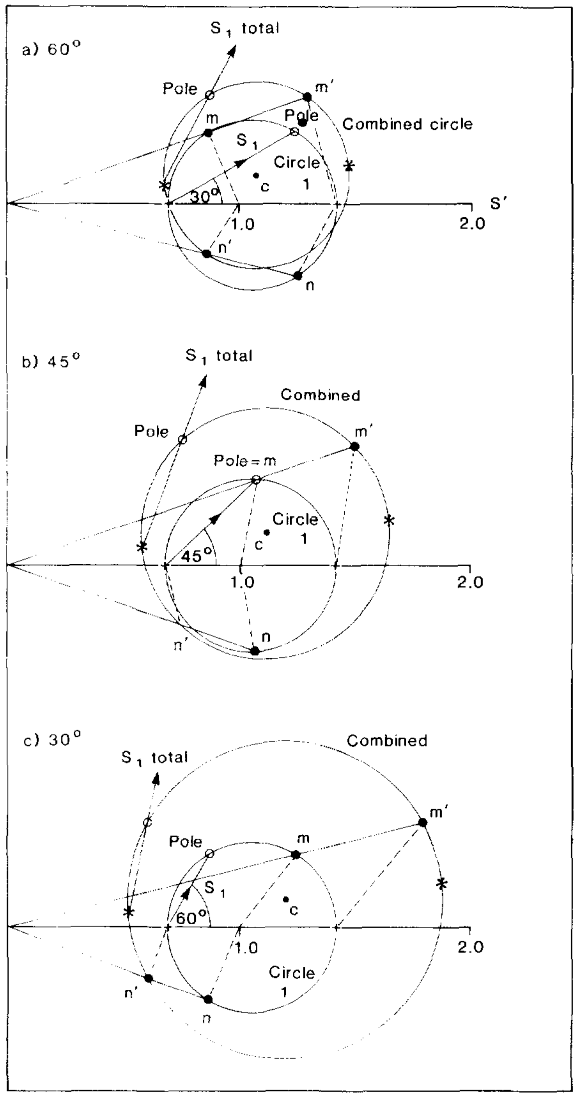

Fig. 2. Two oblique pure shears, of equal strain increment (1.43, 0.7) but in different relative orientations. (a) 60° orientation of principal axes, (b) 45°, and (c) 30°. Construction method as shown in Fig. 1.

The effects of three different intensities of Second Deformation, in the same relative orientation to the First (45°), are shown in Fig. 3 (circles A, B and C). The construction method is the same as shown in Figs. 1 and 2. With increasing intensity of the Second Deformation, the principal stretch directions for the Combined Deformation become quite closely aligned to the directions of Second stretch.

Simple shear followed by pure shear

The principles of superposition of a pure shear parallel to x_1, x_2 (Fig. 1) can be applied to any First Defor-

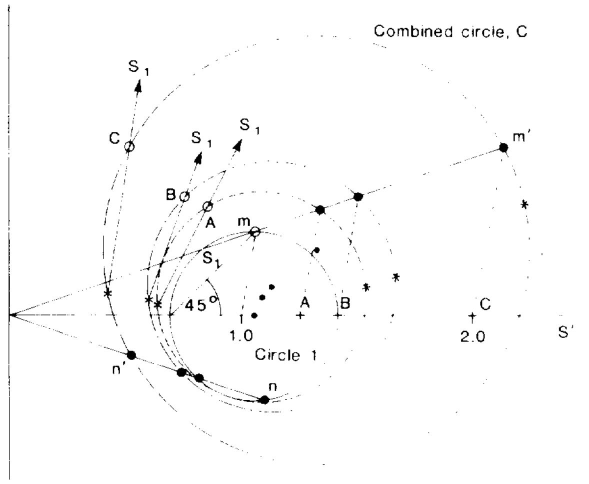

Fig. 3. Two 45°-oblique pure shears, with three different values of Second Deformation: A with principal reciprocal stretches of 1.25 and 0.8; B = 1.43, 0.7; C = 2.0, 0.5. Case B is the same as shown in Fig. 2(b). Case C is the same value of superposed pure shear as in Fig. 1, but in a different orientation. Only the construction lines from Second S_1' are shown (positioning m's), for simplicity. Method as in Fig. 1. Combined Mohr circle for A, small dashes; for B, dot-dashes; for C, large dashes.

mation. This will now be illustrated for a First simple shear deformed by a Second pure shear (Fig. 4). The First Deformation is chosen to be an equivalent strain to the previous examples, for easy comparison; however, this was achieved by a simple shear parallel to x_1, with angular shear, $\psi = 36°$. Figure 4(b) is thus the special 'off axis' Mohr diagram for simple shear. The Second Deformation has principal stretches of 2.0 and 0.5, as in Fig. 1.

The pure shear transformations of points m, n to m′, n′ follow the method described earlier, and are shown in Fig. 4(c). The Combined Deformation circle (Fig. 4d), constructed as before, crosses the reference axis at m′ and the new pole: these are the directions of zero total rotation, for this sequence of superposed deformation. This example of a simple shear followed by a pure shear with a principal direction parallel to the shear direction, produces a characteristic family of Mohr circles which fall into De Paor's (1983) class of 'sub-simple-shear'. Comparison will be made, later, with the inverse history: i.e. a combined deformation produced by pure shear followed by simple shear.

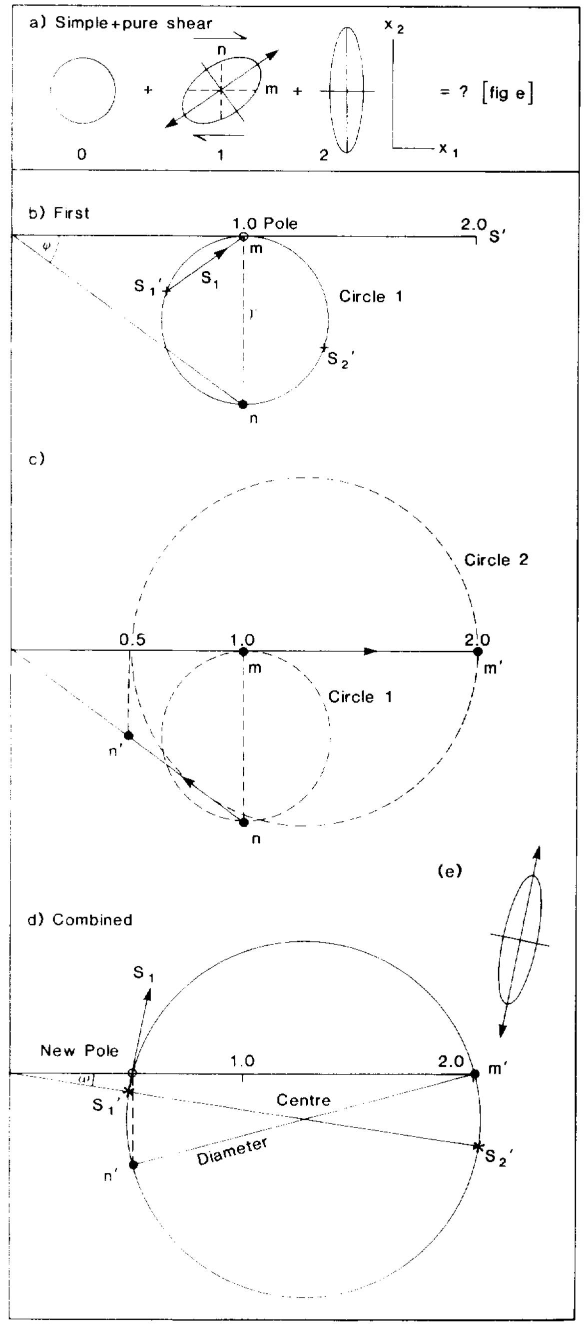

Fig. 4. Construction for simple shear followed by pure shear. For full explanation, see Fig. 1 and text description. Note that (b) is the special 'off-axis' Mohr circle for simple shear.

SUPERPOSED SIMPLE SHEAR

Three types of simple-shear superposition will be considered: pure shear followed by simple shear parallel to one of the pure shear principal directions; pure shear followed by oblique simple shear; and two oblique simple shears. In all cases, the superposed simple shear is parallel to x_1, and as before, the construction depends on the transposition of lines m and n, which were parallel to x_1 and x_2 after the First Deformation. For such a simple-shear transformation, m will neither stretch nor rotate, as it is chosen as the direction of Second simple shear. However, n will rotate by ψ, the angle of shear, and will also stretch. By simple trigonometry, the stretch for the original shear-normal is found to be sec ψ, and so the reciprocal stretch for n′ will be cos ψ. These laws of simple shear transposition of m, n to m′, n′ form the basis of the construction method in the following examples. In summary, whereas pure shear transposition of m, n were two orthogonal stretches and no rotations, simple shear requires no stretch or rotation for m, and a stretch and rotation for n.

Equal-area plane-strain examples will be used, although this is not a requirement of the following methods (except, by definition, for the case of two simple shears).

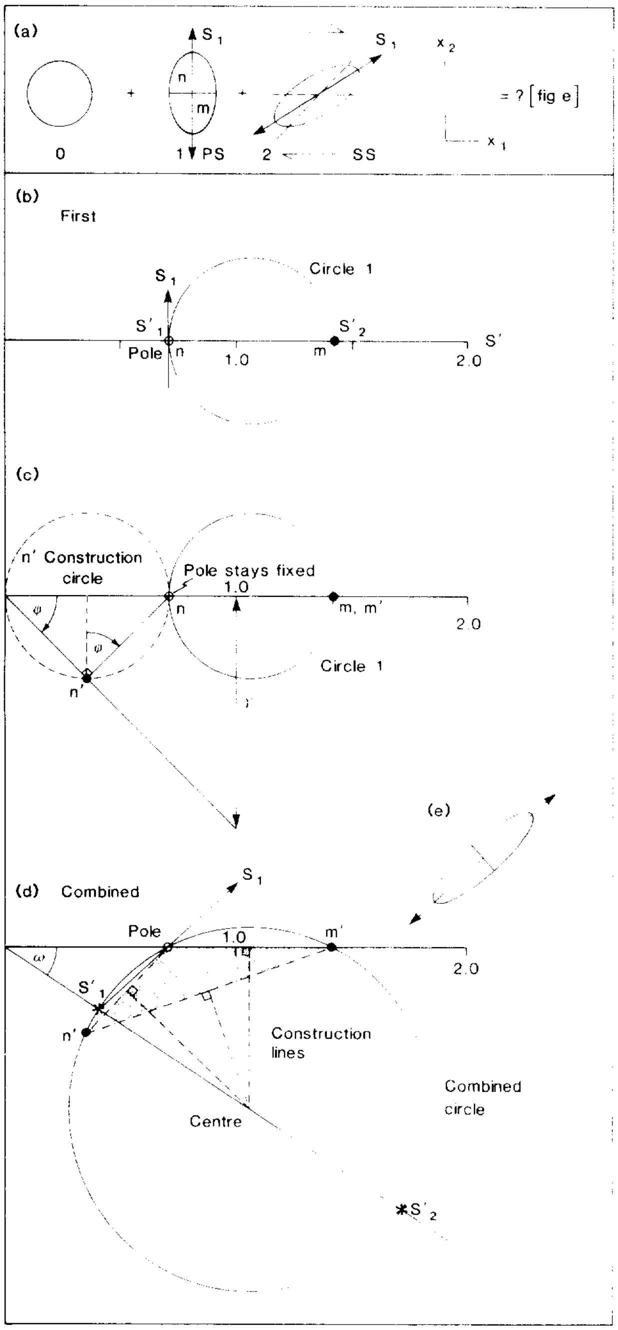

Fig. 5. Simple-shear superposition. This is the special case of a Second simple shear parallel to an axis of a First pure shear (e.g. layer-parallel shortening followed by layer-parallel shearing). (c) shows the n' construction circle used for transposition of n. See text for full description.

Pure shear followed by simple shear parallel to a pure shear axis

The First Deformation is chosen, as before, to be an irrotational pure shear with principal stretches 1.43, 0.7 (Figs. 5a&b). The Second Deformation is a simple shear of $\gamma = 1$ ($\psi = 45°$). The properties of transposition of m, n in the Second simple shear are set out in Fig. 5(c). Point m remains fixed. Point n has been defined above to rotate by ψ (here 45°), and needs to be multiplied by the Second reciprocal stretch of cos ψ. It is found that n moves to n' down the *perpendicular projection* from n to the new ray ($\pm\psi$). From simple geometry, all n' points for different angles of shear are found to fall on a circular arc drawn with the diameter (n–origin) (Fig. 5c). This allows n to be rotated and stretched by cos ψ, in one simple construction. (This n' construction circle should not be confused with a Mohr circle.)

m' and n' are no longer mutually perpendicular, so do not designate the diameter of the Combined circle (unlike the pure-shear superposition examples). However, their geographical orientations are known from the simple shear, which will allow the Mohr circle pole to be positioned for the Combined Deformation. For this example, the n–n' line represents the deformed orientation of n', and its intersection with the horizontal line drawn from m' marks the pole (Fig. 5d). So for this case, the pole remains in its First Deformation position. Three points, m', n' and the pole, define a triangle (shaded in Fig. 5d) which is transcribed by the Combined Mohr circle. The circle can be constructed by the well-known 'bisected chord' method, and the S_1 direction located using the Mohr circle pole. Alternatively, S_1 can be oriented by its angle to m' or n' on the Mohr circle. The resultant deformation for this example (Figs. 5d&e) is very significantly 'off-axis'.

It is clear from comparison with Fig. 4, that the resultant Mohr diagram for pure shear followed by simple shear is very like a simple shear followed by pure shear, as would be expected (Coward & Kim 1981, Sanderson 1982). The Combined circles both have the property of crossing the reference axis at m' and the pole.

As a history of layer-parallel shortening followed by layer-parallel simple shear is commonly considered in geological deformation sequences, some other examples will be briefly investigated here. Figure 6 takes the First Deformation as before (Fig. 5b), and shows Combined Deformations for 15, 30, 45 and 60° of sinistrally or dextrally superposed simple shear. With the revelation in Fig. 5 that m and the pole will remain fixed points during the simple-shear transposition, all that is needed to determine the final circle is the n' point, which is given by the n' construction circle. These sets of progressively 'inflated' circles with increasing simple shear have centres moving progressively 'off-axis' along an ordinate line from the centre of the First circle. The superposed shear strain, γ, can be read from the point of intersection of the ψ ray on an ordinate 'axis' drawn from the 1.0 (reciprocal stretch) point (Fig. 6).

Circles of the form shown in Figs. 4–6 would be classified as 'sub-simple-shear' deformations (De Paor 1983). It follows that a deformation or reciprocal deformation of this type can be factorized into a pure shear followed by a simple shear, without necessarily implying this was the deformation sequence. The 'pure shear factor' would be determined by the intersection points on the reference axis, and the simple shear by the intersection of the n' construction circle, as shown in Fig. 6. This kind of factorization is commonly used to analyse geological deformation variations, and Mohr diagram representations will be pursued in other work.

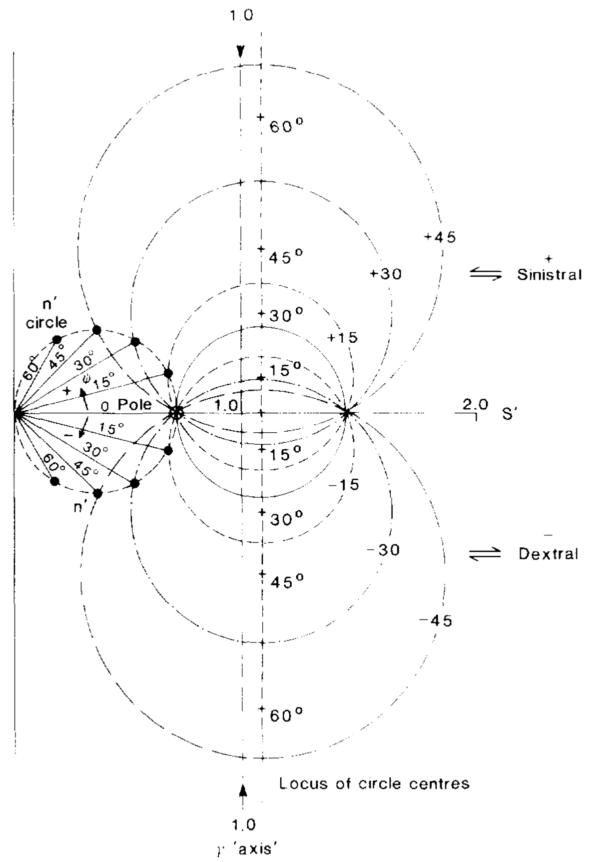

Fig. 6. Set of Mohr circles for Combined Deformation of pure shear followed by simple shear. The n′ construction circle (left) provides the ψ scale. γ, if required, can be read from the intersection of the ψ ray with the ordinate at 1.0 (labelled γ axis). The solid-curve Mohr circle is the First pure shear, and three different broken circles show superposed simple shear of ψ = 15°, 30° and 45° (crosses = circle centres). For ψ = 60°, only n′ and the circle centre are shown. Note that all the circle centres fall on the same ordinate, and pierce the horizontal axis at the same two points (one being the pole).

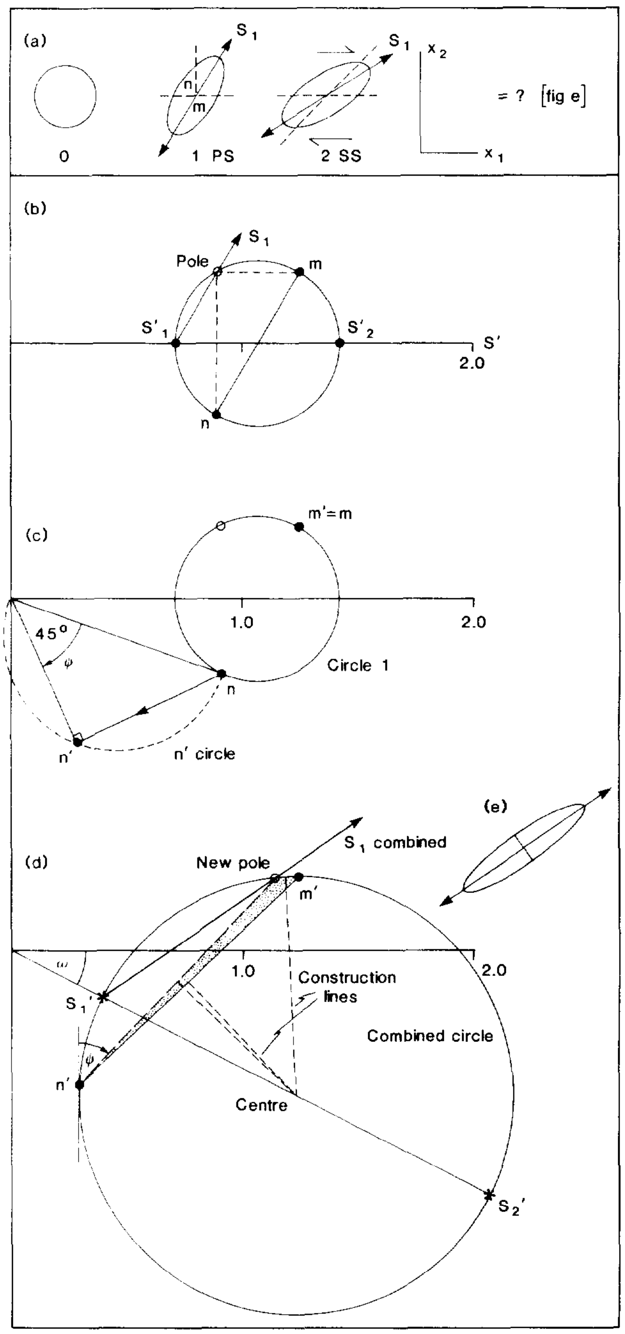

Fig. 7. Simple-shear superposition oblique to the pure-shear axes. Method and notation close to that shown in Fig. 5: see text for details.

Pure shear followed by simple shear oblique to the pure shear axes

The method described above can be applied to a more general case of simple shear superposed upon an earlier pure shear with its principal axes oblique to x_1, x_2 (Fig. 7). The First Deformation has the same value as the previous examples (principal stretches 1.43 and 0.7), at 60° counter-clockwise to x_1, x_2 (Fig. 7b). Points m and n are identified, and drawn with respect to the Mohr circle pole. The transposition shown in Fig. 7(c) is very similar to the method in Fig. 5(c). Point m stays fixed, but in this case, does not lie on the abcissa. Point n′ falls on a circle constructed from n to the origin, as before, but in this case, the circle diameter is an inclined ray.

For this example, the Mohr circle pole does not remain in the same place after the superposition. It is determined by constructing n′ in its geographical orientation (at angle ψ to the vertical in Fig. 7d), and finding the intersection of this line with the horizontal line from m′. The pole, and points m′ and n′, again define a triangle (shaded) which is contained by the Combined Deformation circle. As before, the circle centre is determined by the bisected chord method (see construction lines, Fig. 7d), and the combined strain ellipse is constructed in real space using the pole (Fig. 7e).

Superposed simple shears

This example of two obliquely superposed simple shears follows all the same principles of construction as shown above. The First Deformation has the same strain as in previous examples (1.43, 0.7), envisaged to have occurred by a dextral simple shear with $\psi = 36°$, at 30° to x_1 (Fig. 8a). However, the Second Deformation is here taken to be a *sinistral* simple shear ($\psi = 45°$) parallel to x_1. Thus the First and Second strains are identical to those in Figs. 5 and 7, but the directions differ.

The Combined Deformation determined in Fig. 8

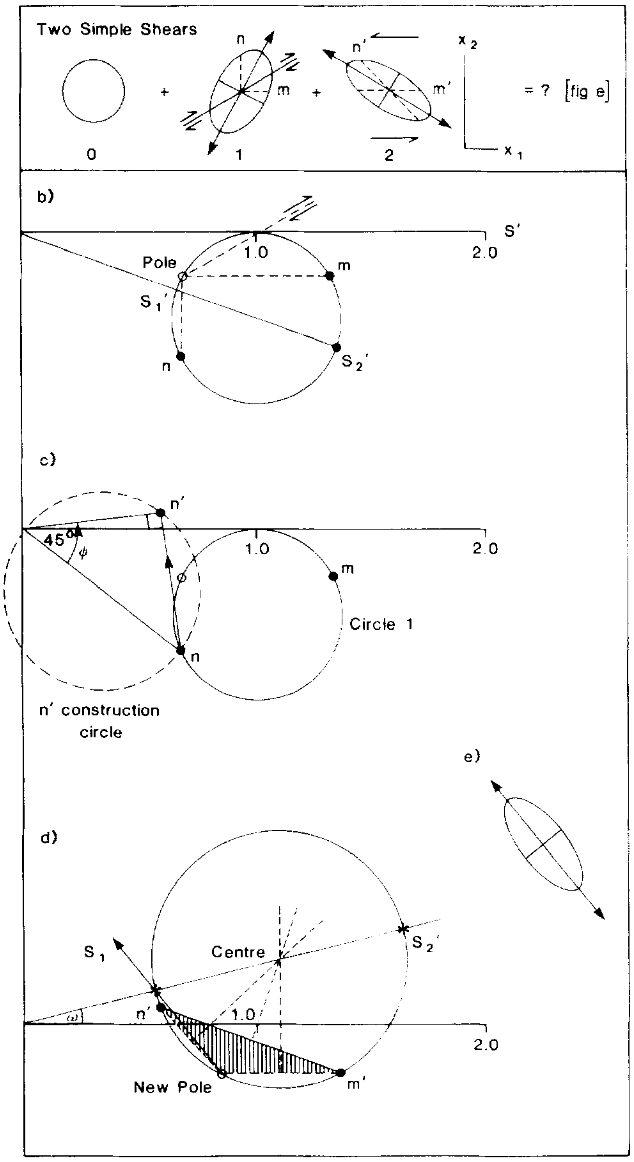

Fig. 8. Two oblique simple shears. The method follows the principles shown in Figs. 5 and 7; see text for other details.

follows the construction principles shown in Fig. 7, so will not be repeated. The only significant difference is the sinistral sense of the Second Deformation, which causes n to move in the opposite sense to before, and to cross the reference axis. The second simple shear is sufficient to result in an overall sinistral sense of rotational deformation.

GENERAL SUPERPOSED DEFORMATION

The methods of superposition developed in the preceding sections and Mohr diagrams, were classified into two types of superposition: superposed pure shear, and superposed simple shear. Examples for a First Deformation of pure shear or simple shear served to illustrate that the procedures could have been applied equally well to any First Deformation. Can they also be applied to any Second Deformation?

To answer this, it is first necessary to recap on how the transposition from one deformed state to another could have been achieved so easily on the Mohr diagram. Both the pure and simple shear superpositions depended on viewing the deformation in terms of external axes (x_1, x_2), with x_1 parallel *either* to a principal axes of superposed pure shear in the material, *or* to the direction of superposed simple shear. In the former, we could transform two lines by stretch and no rotation, so that material lines m, n remained parallel to x_1, x_2 after stretching to m′, n′. In the latter, the transformation was by zero stretch and rotation of m (parallel to x_1), and a simply defined stretch and rotation for n.

In the case of a Second Deformation which is neither pure shear nor simple shear, the principles of superposition and transposition will be more complex. Suppose the Second Deformation were (in reciprocal form) represented by an off-axis Mohr diagram, such as any of the combined deformations derived in the preceding figures. Such circles characterize sub-simple shear, or generalized shear (see Simpson & De Paor 1993). They have two points of zero rotation (the eigenvectors), where the circle crosses the reference axis. From the examples developed in Fig. 6, it is apparent that any such deformation can be factorized into a pure shear on axes given by these eigenvectors, and a simple shear. Thus, a Second Deformation of this type could quite validly be separated into two operations: a pure shear factor, following the transposition procedures shown in Figs. 1–4; and a simple shear transposition, according to the procedures of Figs. 5–8.

It has been stated already that no restrictions are placed on the nature of the First Deformation. It follows that any number of superpositions may be undertaken by Mohr diagram methods, not just two deformations as considered in previous examples. For multiple deformation superpositions, after each transposition of a 'First' by a 'Second', the 'Combined' state must be renamed the First state, and the next increment called the Second: and so on. The method is, of course, restricted to two-dimensional problems. However, it does not require plane strain. For non-plane-strain cases, care must be taken not to equate S_1 with S_2' (or vice versa).

For a fuller understanding of the principles of *superposition of a general deformation*, it is helpful to consider the Second Deformation in both its 'forwards' and 'backwards' state (respectively, **D** and **d**). Recall that all the preceding methodology just used the reciprocal Mohr diagram (**d**). For a general Second Deformation, use of the **D** diagram as well (as in Fig. 9), allows representation of stretches and rotations of material lines in the undeformed state, which is the state of the material at the end of the First Deformation. The stretches and rotations for orthogonal material lines (e.g. m, n) fall on the diagonal of the Second **D** circle, signifying their 90° relationship. On the **d** diagram, however, m and n are represented with respect to their orientations *after* the Second Deformation, so are generally no longer orthogonal. (This matter was side-tracked

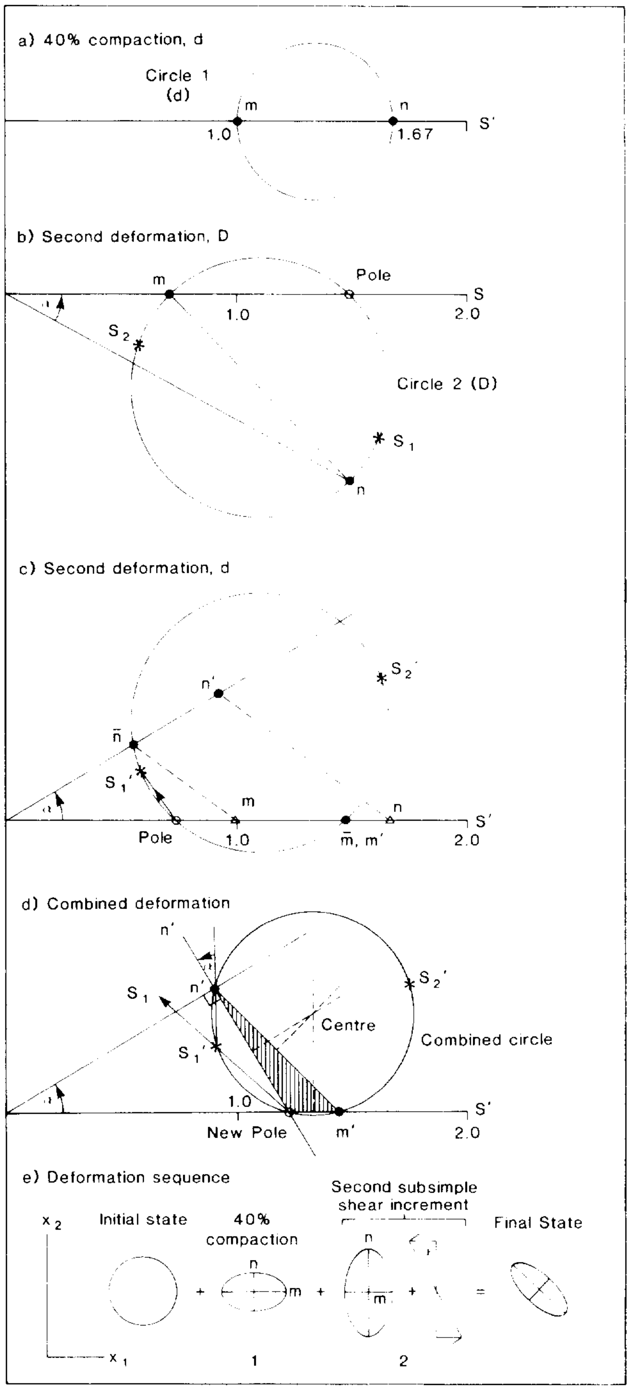

Fig. 9. General superposed deformation by Mohr construction. For full description of procedures, see text. (a) First Deformation of 40% compactional strain. Material line m here represents bedding. (b) Second Deformation of sub-simple shear, in the 'forwards' deformation Mohr representation, **D**. Stretches and rotations shown for m, n. (c) Second Deformation of sub-simple shear: the 'backwards' deformation representation, **d**. Reciprocal stretches and rotations for m, n given by m̄, n̄. Triangles mark the positions of m and n from the First Deformation in (a), transposed by the reciprocal stretches and rotations (m̄, n̄) to m', n'. (d) m' and n' drawn in their geographic positions, in order to determine the new pole. The Combined Deformation Mohr circle is found which encloses the shaded triangle, as before. (e) Summary of the deformation sequence, showing the final state, and the Second Deformation in terms of two components.

in the simple shear examples such as Fig. 5, by using the trigonometric transposition for n'.)

It was noted in the Introduction that the **D** and **d** Mohr diagrams have an inverse relationship. For equal-area plane strain, one is exactly a 'turned over' version of the other. For dilational deformation, however, there is a linear scale change from one to the other.

The principles of general superposition are described by means of the example in Fig. 9. This was chosen to be geologically realistic, and so will illustrate how Mohr diagrams might be used to determine more complex results of deformation superposition in rocks. One problem with real cases is the choice of external axes through which to view the deformation. As discussed by Simpson & De Paor (1993), it may often be legitimate to fix one of the axes to the rock: in their case, the edge of a shear zone; in the present example, a bedding-plane trace. If a progressive deformation associated with folding were represented this way, the x_1 direction would rotate with rotating limbs, so the rotations recorded on the Mohr diagram would omit this component. Bearing this in mind, the sequence of deformation considered in Fig. 9 could be considered either as a compaction followed by a general deformation, according to the lowermost sketch; or a deformation sequence on a fold limb, where the limb rotations are omitted from the treatment.

The First Deformation in Fig. 9 is a compaction of 40% with principal axes parallel to material lines, m, n (with m here taken as the bedding trace). The First Deformation, shown in the reciprocal Mohr diagram in Fig. 9(a), is thus a dilational pure shear. The Second Deformation is chosen from the family of sub-simple-shear (discussed earlier with reference to pure shear followed by simple shear, and vice versa, and illustrated in Fig. 5). It can be considered (Fig. 9e, stage 2) as a pure shear plus a simple shear through angle α. Figure 9(b) is the **D** Mohr diagram for the 'forwards' Second Deformation, not used previously in this paper. As noted above, it allows m and n to be represented as 'undeformed' with respect to this Second Deformation, so falling on the circle diameter. The stretches and rotations of these material lines are shown. The reciprocal Mohr diagram (**d**) for this same Second Deformation is represented in Fig. 9(c). It is an equal-area deformation, so this representation is an exact 'turned over' version of Fig. 9(b). Points m and n in this diagram are distinguished by overbars, to show that these are the positions of m̄ and n̄ *after* the Second Deformation. This diagram is needed to read off the reciprocal stretches for m̄ and n̄, which are then applied by multiplication to the reciprocal stretches for the First Deformation given in Fig. 9(a).

Comparison of Figs. 9 (b) & (c) shows the relationship of undeformed and deformed states. For any material line, such as n, the two Mohr representations must show the same value of rotation (α). It is apparent, then, that the two points where the α ray pierces the circle are the stretch and reciprocal stretch for n. So any point, including the Mohr circle pole, can be related in the undeformed and deformed representations (Figs. 9b&c) by turning over the diagram, and moving to the opposite ray piercement point. For this reason, experienced Mohr diagram users could, with care, work entirely with **d** diagrams, and omit Fig. 9(b) from the sequence (or, as stated in the Introduction, work entirely with the **D** diagram).

The transposition of m and n by the Second Deformation is shown in Fig. 9(c). m is defined not to rotate in the present example, as explained above. Multiplication by $\bar{m}$ moves the point m (bedding trace) to m′. (As m is at 1.0, $\bar{m}$ and m′ are here equivalent.) n will rotate by α, and stretch according to $\bar{n}$, to become n′. Points m′ and n′ are replotted in Fig. 9(d), and drawn in their geographical orientations (m′ horizontal, and n′ at α to vertical). The new pole is thus positioned, and a triangle produced (shaded). The Mohr circle for the Combined Deformation (the total deformation) is drawn according to the chord method, and the total strain ellipse drawn in real space (Fig. 9e). Note that this ellipse is smaller than in all previous examples, because of the initial area loss (compaction).

The summary sketch of this general deformation, given in Fig. 9(e), separates the Second Deformation increment of sub-simple-shear into two components. In doing so, it illustrates how this example of a general deformation could have been considered as *three* superposed deformations: compaction, pure shear and simple shear. This serves as a reminder that any deformation, whether in two or three dimensions, can be factorized into components of dilation, simple shear and pure shear (Ramsay & Huber 1983, p. 47), if body rotations are excluded.

CONCLUSIONS

(1) The effects of superposed deformations in two dimensions can be constructed, using the Mohr diagram for reciprocal stretch and rotation, a representation of the 'backwards deformation' tensor, **d**. The conventions chosen allow the progressively deformed state to be shown graphically, and (by use of the Mohr circle pole) related to their geographic positions.

(2) Two special orthogonal material lines (points in Mohr space) in the First deformed state, which are then transposed by the Second Deformation, are the key to determining the Combined Deformation.

(3) Pure-shear superposition of any earlier deformation involves stretching the two points along their rays, with zero ray rotation.

(4) Simple-shear superposition of any earlier deformation keeps one point fixed, and the other moves by stretch and rotation according to simple shear algebra.

(5) Combinations of superposed pure shear and simple shear, and vice versa, illustrate the potential for using Mohr circles for general shear (sub-simple-shear) for factorization into pure shear and simple shear components.

(6) More complex superpositions of two or more deformations are amenable to Mohr construction methods, although the method is more difficult, particularly if carried out exclusively on the **d** Mohr diagram (or **D** diagram). For this reason, the preferred method uses both forms of Mohr construction.

Acknowledgements—I should like to thank Win Means for his helpful comments and advice during the development of this work, and for copies of his GSA Short Course Manuals which helped my understanding of Mohr diagrams. Richard Hartley is thanked for drawing the diagrams. The comments of the reviewers, Ted Tilley and Simon Wallis, are appreciated.

REFERENCES

Allison, I. 1984. The pole of the Mohr circle. *J. Struct. Geol.* **6**, 331–333.

Choi, C. Y. & Hsü, T. C. 1971. Mohr circles for large and small strains in two-dimensional deformations. *J. Strain Analysis* **6**, 62–69.

Coward, M. & Kim, J. H. 1981. Strain within thrust sheets. In: *Thrust and Nappe Tectonics* (edited by McClay, K. & Price, N. J.). *Spec. Publs geol. Soc. Lond.* **9**, 275–292.

Cutler, J. & Elliott, D. 1983. The compatibility equations and the pole to the Mohr circle. *J. Struct. Geol.* **5**, 287–297.

De Paor, D. G. 1983. Orthographic analysis of geological structures—I. Deformation theory. *J. Struct. Geol.* **5**, 255–277.

De Paor, D. G. & Means, W. D. 1984. Mohr circles of the First and Second Kinds and their use to represent tensor operations. *J. Struct. Geol.* **6**, 693–701.

Means, W. D. 1982. An unfamiliar Mohr circle for finite strain. *Tectonophysics* **89**, T1–T6.

Means, W. D. 1983. Application of the Mohr-circle construction to problems of inhomogeneous deformation. *J. Struct. Geol.* **5**, 279–286.

Means, W. D. 1990. Kinematics, stress, deformation and material behavior. *J. Struct. Geol.* **12**, 953–971.

Means, W. D. 1992. How to do anything with Mohr circles. Short Course Manual, Geological Society of America.

Passchier, C. W. 1988. The use of Mohr circles to describe non-coaxial progressive deformation. *Tectonophysics* **149**, 323–338.

Passchier, C. W. 1990a. Reconstruction of deformed vein sets. *Tectonophysics* **180**, 185–199.

Passchier, C. W. 1990b. A Mohr circle construction to plot the stretch history of material lines. *J. Struct. Geol.* **12**, 513–515.

Passchier, C. W. & Urai, J. L. 1988. Vorticity and strain analysis using Mohr diagrams. *J. Struct. Geol.* **10**, 755–763.

Ramsay, J. G. & Huber, M. 1983. *The Techniques of Modern Structural Geology. Volume 1. Strain Analysis.* Academic Press, London.

Robin, P. 1977. Angular relationships between host and exsolution lamellae and use of the Mohr circle. *Am. Mineral.* **62**, 127–131.

Sanderson, D. J. 1982. Models of strain variations in nappes and thrust sheets: a review. *Tectonophysics* **88**, 201–233.

Simpson, C. & De Paor, D. G. 1993. Strain and kinematic analysis in general shear zones. *J. Struct. Geol.* **15**, 1–20.

Treagus, S. H. 1987. Mohr circles for strain, simplified. *Geol. J.* **22**, 119–132.

Treagus, S. H. 1990. The Mohr diagram for three-dimensional reciprocal stretch vs rotation. *J. Struct. Geol.* **12**, 383–395.

Wallis, S. R. 1992. Vorticity analysis in a metachert from the Sanbagawa Belt, SW Japan. *J. Struct. Geol.* **14**, 271–280.